

Package ‘circhelp’

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Type Package

Title Circular Analyses Helper Functions

Version 1.4.0

Description Light-weight functions for computing descriptive statistics in different circular spaces (e.g., 2pi, 180, or 360 degrees), to handle angle-dependent biases, pad circular data, and more. Specifically aimed for psychologists and neuroscientists analyzing circular data. Basic methods are based on Jammalamadaka and Sen-Gupta (2001) <doi:10.1142/4031>, removal of cardinal biases is based on the approach introduced in van Bergen, Ma, Pratte, & Jehee (2015) <doi:10.1038/nn.4150> and Chetverikov and Jehee (2023) <doi:10.1038/s41467-023-43251-w>.

URL <https://achetverikov.github.io/circhelp/index.html>, <https://github.com/achetverikov/circhelp>

BugReports <https://github.com/achetverikov/circhelp/issues>

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angle_diff_rad	<i>Differences between angles in different circular spaces</i>
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Description

Differences between angles in different circular spaces

Usage

```
angle_diff_rad(a, b)

angle_diff_360(a, b)

angle_diff_180(a, b)

angle_diff_90(a, b)

angle_diff_180_45(a, b)

angle_diff_360_90(a, b)

angle_diff(a, b, period = 360)
```

Arguments

a	first angle
b	second angle
period	the period of the circular space (e.g., 180 for line orientations, 360 for compass directions)

Details

By default, all functions return values in $\pm$ half-range space (e.g., $-\pi$ to π for 2π radian space used by `angle_diff_rad()`) but `angle_diff_180_45()` and `angle_diff_360_90()` return values in $[-1/4 \text{ range}, 3/4 \text{ range}]$ space

Value

difference between a and b

Functions

- `angle_diff_rad()`: angle difference in radians
- `angle_diff_360()`: angle difference in 360 degree space
- `angle_diff_180()`: angle difference in 180 degree space (e.g., line orientation)
- `angle_diff_90()`: angle difference in 90 degree space
- `angle_diff_180_45()`: angle difference in 180 degree space from -45 to 135
- `angle_diff_360_90()`: angle difference in 360 degree space from -90 to 270
- `angle_diff()`: angle difference in a space with an arbitrary period (e.g., 180 or 360)

Examples

```
angle_diff_180(5, 175)
angle_diff_360(5, 175)
angle_diff_90(5, 175)
angle_diff_rad(5, 175)

angle_diff_360(300, 0)
angle_diff_360_90(300, 0)
```

Bae_Luck_2018_data	<i>Data from a motion estimation task</i>
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Description

A dataset with the motion estimation results from Bae & Luck (2018) available from <https://osf.io/download/4m2kb> (some variables are removed, see the link for the full dataset).

Usage

```
Bae_Luck_2018_data
```

Format

A data frame with 20480 rows and 8 variables:

subject_Num observer ID
trial_Num trial number
TargetDirection true motion direction
RespAngle reported motion direction

motionCoh motion coherence

Block block number

Session session number

err estimation error

Source

<https://osf.io/download/4m2kb>

References

Bae, G.-Y., & Luck, S. J. (2018). Decoding motion direction using the topography of sustained ERPs and alpha oscillations. *NeuroImage*, 184(August 2018), 242-255. doi:10.1016/J.NEUROIMAGE.2018.09.029

circ_corr	<i>Circular correlation coefficient</i>
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Description

Computes a circular correlation coefficient as defined in Jammalamadaka & SenGupta (2001).

Usage

```
circ_corr(a, b, ill_defined = FALSE, mu = NULL, na.rm = FALSE)
```

Arguments

a	first variable
b	second variable
ill_defined	is one of the variables mean is not well-defined (e.g., it is uniformly distributed)?
mu	fix the mean parameter of both vectors to a certain value
na.rm	a logical value indicating whether NA values should be removed before the computation proceeds

Value

correlation coefficient

References

Jammalamadaka, S. R., & SenGupta, A. (2001). *Topics in Circular Statistics*. WORLD SCIENTIFIC. doi:10.1142/4031

Examples

```
set.seed(1)
x <- rnorm(1000)
y <- 0.5 * x + sqrt(0.75) * rnorm(1000)
circ_corr(x, y)
```

circ_descr

A set of descriptive statistics for circular data

Description

A set of descriptive statistics for circular data

Usage

```
circ_descr(x, w = NULL, d = NULL, na.rm = FALSE)
```

Arguments

x	vector of angles
w	weights for the values in the vector
d	correction for the bias for data with known spacing
na.rm	a logical value indicating whether NA values should be removed before the computation proceeds

Value

a list with descriptive statistics

- mu - mean
- sigma - standard deviation
- skew_pewsey - skewness as defined by Pewsey
- skew_fischer - skewness as defined by Fischer
- rho - mean resultant length
- skew_rel_to_zero - skewness relative to zero

Examples

```
x <- c(rnorm(50, 0, 0.5), rnorm(20, 1, 0.5))
circ_descr(x)
```

circ_lin_corr

Circular-linear correlation

Description

Implementation of the circular-linear correlation measure introduced by Mardia (1976) and Johnson and Wehrly (1977) as cited in Jammalamadaka & Sengupta (2001).

Usage

```
circ_lin_corr(circ_x, lin_x, na.rm = FALSE)
```

Arguments

<code>circ_x</code>	circular variable
<code>lin_x</code>	linear variable
<code>na.rm</code>	a logical value indicating whether NA values should be removed before the computation proceeds

Details

This measure is computed as

$$r^2 = (r_{xc}^2 + r_{xs}^2 - 2r_{xc}r_{xs}r_{cs}) / (1 - r_{cs}^2)$$

where $r_{xc} = \text{corr}(x, \cos(\alpha))$, $r_{xs} = \text{corr}(x, \sin(\alpha))$, $r_{cs} = \text{corr}(\cos(\alpha), \sin(\alpha))$, and α and x are the circular and linear variables, respectively.

Value

circular-linear correlation measure

References

Jammalamadaka, S. R., & SenGupta, A. (2001). Topics in Circular Statistics. WORLD SCIENTIFIC. doi:10.1142/4031

Examples

```
set.seed(1)
x <- rnorm(50)
a <- rnorm(50, sd = 0.4)
circ_lin_corr(x + a, x)
```

<code>circ_loess</code>	<i>An implementation of circular-linear locally-weighted regression (LOESS)</i>
-------------------------	---

Description

Provides an locally-weighted average when the independent variable is circular and depended variable is linear. Mainly to use with ggplot2.

Usage

```
circ_loess(
  formula = NULL,
  data = NULL,
  angle = NULL,
  y = NULL,
  xseq = NULL,
  circ_space = NULL,
  span = 0.75,
  ...
)
```

Arguments

formula	the formula, e.g., $y \sim x$
data	data to use
angle	a vector of angles (not used if a formula is provided)
y	dependent variable vector (not used if a formula is provided)
xseq	a grid to compute predictions on (optional, the default is to use 500 points spanning the circle)
circ_space	circular space to use (90, 180, 360, or 2π)
span	a span to adjust the degree of smoothing
...	other arguments (ignored)

Details

Weights for the regression are computed as

$$w = (1 - (d/d_{max})^3)^3$$

where d is the angular difference between the point at which the estimate is computed and the angles in the data, and d_{max} is the maximum possible distance. If span is above 1, all points are included and $d_{max} = circ_space / (4 * span)$. Otherwise, a proportion α of the points included based on their distance to the point at which the estimate is computed and d_{max} is the corresponding maximal distance.

Value

an object of `circ_loess` class with the following parameters:

- angle the angles in the data
- y the dependent variable vales in the data
- xseq the grid on which the loess values are estimated
- y_est the estimated loess values
- y_se standard errors
- w weights
- circ_space circular space used
- span span used

See Also

[stats::loess\(\)](#)

Examples

```
library(ggplot2)
p <- ggplot(Pascucci_et_al_2019_data, aes(x = orientation, y = err)) +
  geom_point(alpha = 0.05) +
  labs(x = "Orientation, deg.", y = "Error, deg.")
p1 <- p + geom_smooth(method = "loess") + ggtitle("Standard LOESS")
p2 <- p + geom_smooth(method = "circ_loess", method.args = list(circ_space = 180, span = 0.5)) +
  ggtitle("Circular LOESS, span = 0.5")
p3 <- p + geom_smooth(method = "circ_loess", method.args = list(circ_space = 180, span = 0.2)) +
  ggtitle("Circular LOESS, span = 0.2")
(p1 + p2 + p3)
```

circ_mean_rad	<i>Circular mean</i>
---------------	----------------------

Description

Circular mean

Usage

```
circ_mean_rad(x, na.rm = FALSE)
circ_mean_180(x, na.rm = FALSE)
circ_mean_360(x, na.rm = FALSE)
circ_mean(x, period = 360, na.rm = FALSE)
```

Arguments

x	vector of values
na.rm	a logical value indicating whether NA values should be removed before the computation proceeds
period	the period of the circular space (e.g., 180 for line orientations, 360 for compass directions)

Value

mean of values in the vector

Functions

- `circ_mean_rad()`: circular mean in 2π space
- `circ_mean_180()`: circular mean in 180° space (e.g., line orientation)
- `circ_mean_360()`: circular mean in 360° space
- `circ_mean()`: circular mean in a space with an arbitrary period (e.g., 180 or 360)

Examples

```
x <- runif(1000, -pi, pi)
mean(x)
circ_mean_rad(x)
```

circ_sd_rad	<i>Circular standard deviation</i>
-------------	------------------------------------

Description

Circular standard deviation

Usage

```
circ_sd_rad(x, na.rm = FALSE)

circ_sd_360(x, na.rm = FALSE)

circ_sd_180(x, na.rm = FALSE)

circ_sd(x, period = 360, na.rm = FALSE)
```

Arguments

x	vector of angles
na.rm	a logical value indicating whether NA values should be removed before the computation proceeds
period	the period of the circular space (e.g., 180 for line orientations, 360 for compass directions)

Value

standard deviation of values in the vector

Functions

- `circ_sd_rad()`: SD of angles in radians
- `circ_sd_360()`: SD of angles in 360 degree space
- `circ_sd_180()`: SD of angles in 180 degree space
- `circ_sd()`: SD of angles in a space with an arbitrary period (e.g., 180 or 360)

Examples

```
circ_sd_rad(rnorm(50))
circ_sd_180(rnorm(50))
```

correct_angle_rad	<i>Get angle value in $[-\pi, \pi]$ space</i>
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Description

Get angle value in $[-\pi, \pi]$ space

Usage

```
correct_angle_rad(x)
```

Arguments

x	angle
---	-------

Value

angle in $[-\pi, \pi]$ space

Examples

```
correct_angle_rad(4 * pi)
```

density_asymmetry	<i>Compute asymmetry in weighted probability density</i>
-------------------	--

Description

This function calculates the asymmetry in the probability density of a given variable (usually errors) relative to another variable (usually dissimilarity) using kernel density estimation. The asymmetry is computed for each x-axis value, and the result can be averaged or returned for each value individually.

Usage

```
density_asymmetry(
  dt,
  circ_space = 180,
  weights_sd = 10,
  kernel_bw = NULL,
  xvar = "abs_td_dist",
  yvar = "bias_to_distr_corr",
  by = c(),
  n = 181,
  average = T,
  return_full_density = F,
  normalize = T,
  wrap = TRUE,
  exclude_antipode = TRUE,
```

```

    n_wraps = 1,
    rescale_narrow = TRUE,
    scale_safety = 0.5
  )

```

Arguments

<code>dt</code>	data.table with the data.
<code>circ_space</code>	Circular space, which can be 180 or 360 (default: 180).
<code>weights_sd</code>	Standard deviation of the Gaussian window to use across <code>xvar</code> (default: 10).
<code>kernel_bw</code>	Bandwidth for the kernel density estimator across <code>yvar</code> . If NULL, it is computed using <code>stats::bw.SJ()</code> . If "average", it is computed using <code>stats::bw.SJ()</code> for each group indicated by <code>by</code> and then averaged. Otherwise it is passed to <code>stats::density()</code> (default: NULL).
<code>xvar</code>	X-axis variable, such as dissimilarity between items (default: "abs_td_dist").
<code>yvar</code>	Y-axis variable, normally errors (default: "bias_to_distr_corr").
<code>by</code>	A vector of grouping variable names (default: an empty vector).
<code>n</code>	The number of steps for the x-axis variable at which the density is computed (default: 181).
<code>average</code>	If TRUE, the asymmetry is averaged for each x-value (default: TRUE).
<code>return_full_density</code>	If TRUE, returns the full data.table with density computed at each point (default: FALSE).
<code>normalize</code>	if TRUE, normalizes the difference in probability density by the total sum of probability density (with zero point excluded). Delta then corresponds to the probability of observing a given sign. In use only when <code>average</code> is TRUE.
<code>wrap</code>	If TRUE (default), the kernel density estimate is wrapped around the circle, so that a kernel centred near one end of the <code>yvar</code> axis reappears at the other end instead of being truncated. Set to FALSE for the pre-1.4.0 (truncated) behaviour. On orientation or colour data, where errors sit near zero and the bandwidth is a few degrees, this changes nothing to $\sim 1e-13$; it matters when <code>yvar</code> carries mass near $\pm \text{circ_space}/2$, such as the 180-degrees-off reversals in motion-direction data.
<code>exclude_antipode</code>	If TRUE (default), the $\pm \text{circ_space}/2$ point is dropped from the positive and negative sums, exactly like zero. It is the same angle approached from two sides and so has no sign, and with <code>n</code> odd it is otherwise counted twice (once as $-\text{max_diss}$ and once as $+\text{max_diss}$). Set to FALSE for the pre-1.4.0 behaviour.
<code>n_wraps</code>	Number of periodic images summed on each side when <code>wrap</code> is TRUE (default: 1). One is already far past double precision for any bandwidth much smaller than <code>circ_space</code> .
<code>rescale_narrow</code>	If TRUE (default), a bandwidth narrower than the spacing of the <code>x_grid</code> triggers a rescaling of <code>yvar</code> and the bandwidth by a common factor, so that the kernel is at least one grid cell wide. The asymmetry is a signed mass difference and a positive scaling preserves each value's sign, so the estimand is unchanged; this is equivalent to evaluating on a finer grid, but cheaper. Without it, a kernel narrower than a cell falls between grid points and the rectangle sum stops representing the density (at <code>kernel_bw = dx/20</code> a point centred on a cell contributes 7.98x its mass and one half a cell away contributes 0). <code>stats::bw.SJ()</code> does reach that regime on real data. Set to FALSE for the pre-1.4.0 behaviour.

`scale_safety` Fraction of `circ_space/2` that the rescaled `yvar` values are allowed to reach (default: 0.5). Rescaling treats the circular axis as linear, which is only valid while no mass is near the wrap, so the factor is capped rather than assumed safe.

Value

A data.table with the grouping variables, `dist` - the values of X-axis variable at which the density is computed, and `delta` - the difference (asymmetry) in probability density for positive and negative values of `yvar`; or the full density data if `return_full_density` is TRUE.

Examples

```
data(Pascucci_et_al_2019_data)
ex_data <- Pascucci_et_al_2019_data
ex_data[, err := angle_diff_180(reported, orientation)] # response errors
ex_data[, prev_ori := shift(orientation), by = observer] # orientation on previous trial

# determine the shift in orientations between trials
ex_data[, diff_in_ori := angle_diff_180(prev_ori, orientation)]
ex_data[, abs_diff_in_ori := abs(diff_in_ori)]
ex_data[, err_rel_to_prev_targ := ifelse(diff_in_ori < 0, -err, err)]

err_dens <- density_asymmetry(ex_data[!is.na(err_rel_to_prev_targ)],
  circ_space = 180, weights_sd = 10, xvar = "abs_diff_in_ori",
  yvar = "err_rel_to_prev_targ", by = c("observer")
)

library(ggplot2)
ggplot(err_dens, aes(x = dist, y = delta)) +
  geom_line(stat = "summary", fun = mean) +
  labs(y = "Asymmetry in error probability density, %", x = "Absolute orientation difference, °")
```

density_asymmetry_discrete

Compute asymmetry in weighted probability density for discrete data

Description

This function calculates the asymmetry in the probability density of a given variable using kernel density estimation. Unlike `density_asymmetry`, it does not take an `xvar` and assumes that the asymmetry is calculated for the whole dataset or subsets defined by `by` (which could also include a discrete `x` variable if needed).

Usage

```
density_asymmetry_discrete(
  dt,
  yvar = "bias_to_distr_corr",
  circ_space = 180,
  kernel_bw = NULL,
  by = c(),
  n = 181,
```

```

    average = T,
    return_full_density = F,
    normalize = T,
    wrap = TRUE,
    exclude_antipode = TRUE,
    n_wraps = 1,
    rescale_narrow = TRUE,
    scale_safety = 0.5
  )

```

Arguments

dt	data.table with the data.
yvar	Y-axis variable, normally errors (default: "bias_to_distr_corr").
circ_space	Circular space, which can be 180 or 360 (default: 180).
kernel_bw	Bandwidth for the kernel density estimator across yvar. If NULL, it is computed using <code>stats::bw.SJ()</code> . If "average", it is computed using <code>stats::bw.SJ()</code> for each group indicated by by and then averaged. Otherwise it is passed to <code>stats::density()</code> (default: NULL).
by	A vector of grouping variable names (default: an empty vector).
n	The number of steps for the density computation (default: 181).
average	If TRUE, the asymmetry is averaged (default: TRUE).
return_full_density	If TRUE, returns the full data.table with density computed at each point (default: FALSE).
normalize	if TRUE, normalizes the difference in probability density by the total sum of probability density (with zero point excluded). Delta then corresponds to the probability of observing a given sign. In use only when average is TRUE.
wrap	If TRUE (default), the kernel density estimate is wrapped around the circle, so that a kernel centred near one end of the yvar axis reappears at the other end instead of being truncated. See <code>density_asymmetry()</code> .
exclude_antipode	If TRUE (default), the +/- circ_space/2 point is dropped from the positive and negative sums, exactly like zero. See <code>density_asymmetry()</code> .
n_wraps	Number of periodic images summed on each side when wrap is TRUE (default: 1).
rescale_narrow	If TRUE (default), a bandwidth narrower than the density grid spacing triggers a rescaling of yvar and the bandwidth by a common factor. See <code>density_asymmetry()</code> .
scale_safety	Fraction of circ_space/2 that the rescaled yvar values are allowed to reach (default: 0.5).

Value

A data.table with the grouping variables and delta - the difference (asymmetry) in probability density for positive and negative values of yvar; or the full density data if return_full_density is TRUE.

Examples

```

data(Pascucci_et_al_2019_data)
ex_data <- Pascucci_et_al_2019_data
ex_data[, err := angle_diff_180(reported, orientation)] # response errors
ex_data[, prev_ori := shift(orientation), by = observer] # orientation on previous trial

# determine the shift in orientations between trials
ex_data[, diff_in_ori := angle_diff_180(prev_ori, orientation)]
ex_data[, abs_diff_in_ori := abs(diff_in_ori)]
ex_data[, err_rel_to_prev_targ := ifelse(diff_in_ori < 0, -err, err)]
ex_data[, similarity_discrete := ifelse(abs_diff_in_ori > 45, 'Dissimilar', 'Similar')]
err_dens_discrete <- density_asymmetry_discrete(ex_data[!is.na(err_rel_to_prev_targ)],
  circ_space = 180, yvar = "err_rel_to_prev_targ", by = c("observer", "similarity_discrete")
)

library(ggplot2)
ggplot(err_dens_discrete, aes(x = similarity_discrete, y = delta)) +
  geom_violin() + geom_point() +
  labs(y = "Asymmetry in error probability density, %", x = "Previous target")

```

pad_circ

*Pad circular data on both ends***Description**

Pad circular data on both ends

Usage

```

pad_circ(
  data,
  circ_var,
  circ_borders = c(-90, 90),
  circ_part = 1/6,
  verbose = FALSE
)

```

Arguments

data	data.table to pad
circ_var	circular variable
circ_borders	range of the circular variable
circ_part	padding proportion
verbose	print extra info

Details

Pads the data by adding a part of the data (default: 1/6th) from one end to another end. Useful to roughly account for circularity when using non-circular methods.

Value

a padded data.table

Examples

```
dt <- data.table::data.table(x = runif(1000, -90, 90), y = rnorm(1000))  
pad_circ(dt, "x", verbose = TRUE)
```

Pascucci_et_al_2019_data

Data from an orientation estimation task

Description

A dataset with the orientation estimation results from Experiment 2 in Pascucci et al. (2019) available from <https://doi.org/10.5281/zenodo.2544946>.

Usage

Pascucci_et_al_2019_data

Format

A data frame with 4400 rows and 5 variables:

observer observer ID

orientation true orientation

reported reported orientation

rt response time

err estimation error

Source

https://zenodo.org/record/2544946/files/Experiment2_rawdata.csv?download=1

References

Pascucci, D., Mancuso, G., Santandrea, E., Libera, C. D., Plomp, G., & Chelazzi, L. (2019). Laws of concatenated perception: Vision goes for novelty, decisions for perseverance. PLoS Biology, 17(3). [doi:10.1371/journal.pbio.3000144](https://doi.org/10.1371/journal.pbio.3000144)

remove_cardinal_biases

Remove cardinal biases

Description

Remove cardinal biases

Usage

```
remove_cardinal_biases(
  err,
  x,
  space = "180",
  bias_type = "fit",
  plots = "hide",
  poly_deg = 4,
  var_sigma = TRUE,
  var_sigma_poly_deg = 4,
  reassign_at_boundaries = TRUE,
  reassign_range = 2,
  break_points = NULL,
  init_outliers = NULL,
  debug = FALSE,
  do_plots = NULL
)
```

Arguments

err	a vector of errors, deviations of response from the true stimuli
x	a vector of true stimuli in degrees (see space)
space	circular space to use (a string: 180 or 360)
bias_type	bias type to use (fit, card, obl, or custom, see details)
plots	a string hide, show, or return to hide, show, or return plots (default: hide)
poly_deg	degree of the fitted polynomials for each bin (default: 4)
var_sigma	allow standard deviation (width) of the fitted response distribution to vary as a function of distance to the nearest cardinal (default: True)
var_sigma_poly_deg	degree of the fitted polynomials for each bin for the first approximation for the response distribution to select the best fitting model (default: 4)
reassign_at_boundaries	select the bin for the observations at the boundaries between bins based on the best-fitting polynomial (default: True)
reassign_range	maximum distance to the boundary at which reassignment can occur (default: 2 degrees)
break_points	can be used to assign custom break points instead of cardinal/oblique ones with bias_type set to custom (default: NULL)

init_outliers	a vector determining which errors are initially assumed to be outliers (default: NULL)
debug	print some extra info (default: False)
do_plots	deprecated, use the parameter plots instead

Details

If the `bias_type` is set to `fit`, the function computes the cardinal biases in the following way:

1. Create two sets of bins, splitting the stimuli vector into bins centered at cardinal and at oblique directions.
2. For each set of bins, fit a penalised spline (P-spline — a regression spline with a roughness penalty that controls smoothness) for the mean response in each bin. Optionally (see `var_sigma`), the response variability (SD) is modelled jointly via a log-linear model, allowing the response distribution to vary in width as a function of distance to the nearest cardinal (regardless of whether the bins are centered at the cardinal or at the oblique, the width of the response distribution usually increases as the distance to cardinals increases).
3. Choose the best-fitting model between the one using cardinal and the one using oblique bins.
4. Optionally (see `reassign_at_boundaries`), reassign observations near bin boundaries to the bin whose fitted model best describes them, iterating until convergence.
5. Compute the residuals of the best-fitting model - that's your bias-corrected error - and the biases (see below).

The bias is computed by flipping the sign of errors when the average predicted error is negative, so, that, for example, if on average the responses are shifted clockwise relative to the true values, the trial-by-trial error would count as bias when it is also shifted clockwise.

If `bias_type` is set to `obl` or `card`, only one set of bins is used, centred at cardinal or oblique angles, respectively.

For additional examples see the help vignette: `vignette("cardinal_biases", package = "circular")`

Value

If `plots== 'return'`, returns the three plots showing the biases (combined together with `patchwork::wrap_plots()`). Otherwise, returns a list with the following elements:

- `is_outlier` - 0 for outliers (defined as $\pm 3 \times \text{pred_sigma}$ for the model with varying sigma or as $\pm 3 \times \text{SD}$ for the simple model)
- `pred` predicted error
- `be_c` error corrected for biases (`be_c` = observed error - pred)
- `which_bin` the numeric ID of the bin that the stimulus belong to
- `bias` the bias computed as described above
- `bias_typ` bias type (cardinal or oblique)
- `pred_lin` predicted error for a simple linear model for comparison
- `pred_sigma` predicted SD of the error distribution
- `coef_sigma_int`, `coef_sigma_slope` intercept and slope for the sigma prediction

References

- Chetverikov, A., & Jehee, J. F. M. (2023). Motion direction is represented as a bimodal probability distribution in the human visual cortex. *Nature Communications*, 14(7634). doi:10.1038/s4146702343251w
- van Bergen, R. S., Ma, W. J., Pratte, M. S., & Jehee, J. F. M. (2015). Sensory uncertainty decoded from visual cortex predicts behavior. *Nature Neuroscience*, 18(12), 1728-1730. doi:10.1038/nn.4150

Examples

```
# Data in orientation domain from Pascucci et al. (2019, PLOS Bio),
# https://doi.org/10.5281/zenodo.2544946

ex_data <- Pascucci_et_al_2019_data[observer == 4, ]
remove_cardinal_biases(ex_data$err, ex_data$orientation, plots = "show")

# Data in motion domain from Bae & Luck (2018, Neuroimage),
# https://osf.io/2h6w9/
ex_data_bae <- Bae_Luck_2018_data[subject_Num == unique(subject_Num)[5], ]
remove_cardinal_biases(ex_data_bae$err, ex_data_bae$TargetDirection,
  space = "360", plots = "show"
)

# Using a stricter initial outlier boundary

remove_cardinal_biases(ex_data_bae$err, ex_data_bae$TargetDirection,
  space = "360", plots = "show",
  init_outliers = abs(ex_data_bae$err) > 60
)

# We can also use just one bin by setting `bias_type` to custom
# and setting the `break_points` at the ends of the range for x

remove_cardinal_biases(ex_data_bae$err, ex_data_bae$TargetDirection,
  space = "360", bias_type = "custom",
  break_points = c(-180, 180), plots = "show",
  reassign_at_boundaries = FALSE, poly_deg = 8,
  init_outliers = abs(ex_data_bae$err) > 60
)
```

```
remove_cardinal_biases_discrete
```

Remove cardinal biases for data with orientation (color, motion, ...) set in discrete steps

Description

Remove cardinal biases for data with orientation (color, motion, ...) set in discrete steps

Usage

```
remove_cardinal_biases_discrete(err, x, space, init_outliers = NULL)
```

Arguments

err	a vector of errors, deviations of response from the true stimuli
x	a vector of true stimuli in degrees (see space)
space	circular space to use (a string: 180 or 360)
init_outliers	a vector determining which errors are initially assumed to be outliers (default: NULL)

Value

returns a data.table with the following columns:

- is_outlier - 0 for outliers (defined as $\pm 3 \times$ predicted SD, where SD and mean are computed after excluding initially assumed outliers)
- be_c error corrected for biases (be_c = observed error - pred)

smoothed_circ_sd	<i>Compute smoothed estimate of circular standard deviation</i>
------------------	---

Description

This function calculates smoothed estimates of circular standard deviation using weighted kernel density estimation. It works similarly to density_asymmetry by applying Gaussian weights across an x-axis variable to compute localized circular SD estimates.

Usage

```
smoothed_circ_sd(
  dt,
  circ_space = 180,
  weights_sd = 10,
  xvar = "abs_td_dist",
  yvar = "err",
  by = c(),
  n_points = 91
)
```

Arguments

dt	data.table with the data.
circ_space	Circular space, which can be 180 or 360 (default: 180).
weights_sd	Standard deviation of the Gaussian window to use across xvar (default: 10).
xvar	X-axis variable, such as dissimilarity between items (default: "abs_td_dist").
yvar	Y-axis variable, normally errors (default: "err").
by	A vector of grouping variable names (default: an empty vector).
n_points	The number of points along the x-axis at which to compute circular SD (default: 91).

Value

A data.table with the grouping variables, dist - the values of X-axis variable at which the circular SD is computed, and circ_sd - the smoothed circular standard deviation estimate at each point.

Examples

```
data(Pascucci_et_al_2019_data)
ex_data <- Pascucci_et_al_2019_data
ex_data[, err := angle_diff_180(reported, orientation)] # response errors
ex_data[, prev_ori := shift(orientation), by = observer] # orientation on previous trial

# determine the shift in orientations between trials
ex_data[, diff_in_ori := angle_diff_180(prev_ori, orientation)]
ex_data[, abs_diff_in_ori := abs(diff_in_ori)]

circ_sd_smooth <- smoothed_circ_sd(ex_data[!is.na(diff_in_ori)],
  circ_space = 180, weights_sd = 10, xvar = "abs_diff_in_ori",
  yvar = "err", by = c("observer")
)

library(ggplot2)
ggplot(circ_sd_smooth, aes(x = dist, y = circ_sd)) +
  geom_line(stat = "summary", fun = mean) +
  labs(y = "Circular SD, °", x = "Absolute orientation difference, °")
```

vm_kappa_to_circ_sd	<i>Conversion between the circular SD and kappa of von Mises</i>
---------------------	--

Description

Conversion between the circular SD and kappa of von Mises

Usage

```
vm_kappa_to_circ_sd(kappa)

vm_kappa_to_circ_sd_deg(kappa)

vm_circ_sd_to_kappa(sd)

vm_circ_sd_deg_to_kappa(sd_deg)
```

Arguments

kappa	von Mises kappa parameter
sd	circular SD of von Mises (radians)
sd_deg	circular SD of von Mises (degrees)

Value

vm_kappa_to_circ_sd and vm_kappa_to_circ_sd_deg return circular SD (in radians or degrees, respectively) corresponding to a given kappa. vm_circ_sd_to_kappa and vm_circ_sd_deg_to_kappa return kappa corresponding to a given circular SD (in radians or degrees, respectively).

Functions

- vm_kappa_to_circ_sd_deg(): get circular SD (in degrees) from kappa
- vm_circ_sd_to_kappa(): get kappa from circular SD (in radians)
- vm_circ_sd_deg_to_kappa(): get kappa from circular SD (in degrees)

Examples

```
vm_kappa <- 5
vm_sd <- vm_kappa_to_circ_sd(vm_kappa)

vm_circ_sd_to_kappa(vm_sd)

vm_kappa_to_circ_sd(vm_circ_sd_to_kappa(vm_sd))
```

weighted_circ_mean	<i>Weighted circular parameters</i>
--------------------	-------------------------------------

Description

Weighted circular parameters

Usage

```
weighted_circ_mean(x, w, period = NULL, na.rm = FALSE)

weighted_circ_mean2(x, w, na.rm = FALSE)

weighted_circ_sd(x, w, na.rm = FALSE)

weighted_circ_rho(x, w, na.rm = FALSE)
```

Arguments

x	vector of values (in radians, unless period is set)
w	vector of weights
period	if not NULL, the period of the circular space (e.g., 180 or 360); x is then converted to radians as $x / \text{period} * 2 * \pi$ and the result is converted back to the same period
na.rm	a logical value indicating whether NA values should be removed before the computation proceeds

Value

weighted mean of values in the vector

Functions

- `weighted_circ_mean()`: weighted circular mean
- `weighted_circ_mean2()`: an alternative way to compute weighted circular mean (the results are the same)
- `weighted_circ_sd()`: weighted circular SD
- `weighted_circ_rho()`: weighted mean resultant length

Examples

```
x <- rnorm(1000, 0, 0.5)
w <- runif(1000, 0, 1)
weighted.mean(x, w)
weighted_circ_mean(x, w)
```

weighted_sem	<i>Weighted standard error of the mean (SEM_w)</i>
--------------	--

Description

Computes the variance of a weighted mean following the definitions given by Kirchner (2006).

Usage

```
weighted_sem(x, w, na.rm = FALSE)
```

Arguments

x	variable to compute the SEM for
w	weights
na.rm	should NAs be removed

Details

James Kirchner describes two different cases when the weighted variance is computed. The code here implements Case I where "one wants to give more weight to some points than to others, because they are considered to be more important" and "the weights differ but the uncertainties associated with the individual x_i are assumed to be the same" (Kirchner, 2006, p. 1). The formula used is:

$$SEM_w = \sqrt{\left(\sum_{i=1}^N (w_i x_i^2) - \bar{x}^2 \right) \frac{\sum_{i=1}^N w_i^2}{1 - \sum_{i=1}^N w_i^2}}$$

The expected error is within 5% of the bootstrapped SEM (at larger sample sizes).

Value

weighted standard error of the mean

References

- Kirchner, J. 2006. Data Analysis Toolkit #12: Weighted averages and their uncertainties. https://seismo.berkeley.edu/~kirchner/Toolkits/Toolkit_12.pdf. Retrieved on 04.07.2024.
- Bevington, P. R. 1969. Data Reduction and Error Analysis for the Physical Sciences. McGraw-Hill, 336 pp.

Examples

```
set.seed(1)
n_obs <- 200
w <- runif(n_obs)
w <- w / sum(w)
x <- rnorm(n_obs, sd = 5)
weighted_sem(x, w)
```

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